Technical Comments

Comment on "A Direct Matrix Method for the Divergence Problem"

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IT is curiously refreshing to be reminded from time to time of the struggle that accompanies the advancing state-of-the-art of the aeronautical sciences. Lest we take our present position too much for granted, Ref. 1 has just appeared from out of the past to recall the period of the 1950's during which the field of static aeroelasticity made the transition from the slide-rule to the high-speed digital computer. The following historical recollections are offered as a more complete review of that dynamic period of progress.

Equation (4) of Ref. (1) was almost implicit in Eq. (8–55) of Bisplinghoff, Ashley, and Halfman.[2] The definition of static and oscillatory aerodynamic influence coefficients (AIC's) in Ref. 3 [Eq. (6) of Ref. 1 appears as Eq. (8) in Ref. 3] provided for a general formulation of the various aeroelastic problems which was outlined in the introduction to Ref. 4. A general purpose computer program was developed in Ref. 5 for the static aeroelastic analysis of the problems of rigid and flexible load distributions, divergence, estimation of rigid and flexible static and dynamic stability derivatives, and the correction of wind-tunnel data measured on flexible models. Both structural influence coefficients and AIC's were used and Eq. (4) of Ref. 1 is found as Eq. (23) in Ref. 5.

The example problem used to demonstrate the computer program of Ref. 5 was also the jet transport wing of Bisplinghoff, Ashley, and Halfman.[2] However, the AIC's were based on the subsonic lifting surface theory (at a Mach number of zero) of Runyan and Woolston.[6,7] The first two divergence dynamic pressures were calculated to illustrate the divergence option, and were $q_1 = 3786$ and $q_2 = 26{,}951$ psf. The fundamental divergence dynamic pressure corresponds to the velocity $V_1 = 1784$ fps at sea level, with which the value in Ref. 1 using lifting line theory agrees well.

Later calculations of the divergence characteristics of the jet transport wing were performed using incompressible strip theory in connection with a method for transient flutter analysis.[8] The five divergence dynamic pressures were found to be $q_1 = 2397$, $q_2 = 9510$, $q_3 = 23{,}555$, $q_4 = 38{,}208$, and $q_5 = 69{,}914$ psf. The fundamental divergence velocity at sea level is $V_1 = 1420$ fps, as was mentioned in Ref. 8. If the aspect ratio correction used in Refs. 1 and 2 is made, the divergence velocity at sea level becomes $V_1 = 1950$ fps and agrees with the result in Ref. 2, p. 440.

The concern in Ref. 1 over the null eigenvalues from bending is unwarranted. The five nonzero eigenvalues of the example wing (using strip theory) were found easily by the power method because of their wide separation. The sixth through tenth eigenvalues were not zero (infinite divergence speeds) because of round-off errors in the eigen-matrix deflation required in the power method; however, the sixth eigenvalue was four orders of magnitude lower than the fifth and thereby gave some clue to what should have been expected.

References

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Reply by Author to William P. Rodden

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THE author wishes to thank Dr. Rodden for his comments. One of the objects of the Note was to draw attention to the procedure (which the author believes to be superior to the assumed mode method both for torsional and chordwise divergence calculations). But the main aim was to point out a method of overcoming the difficulties associated with the zero eigenvalues arising from bending.

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Comments on "Synthesis of Tire Equations for Use in Shimmy and Other Dynamic Studies"

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THE study by Rogers and Brewer[1] proposes an empirical determination of the differential equations governing tire motion for aircraft shimmy with the central idea that their method is accurate and practical while other methods are either inaccurate or impractical.

In particular they cite the 5 parameter modified Moreland theory of Ref. 2 and 3 as being inaccurate because it does not agree with their tire moment vs reduced frequency curves at higher values of reduced frequency. Since the modified Moreland method has been used successfully in many cases and is more straightforward than the method of Rogers and Brewer, it is felt that some clarification of the importance of the short wavelength response of a tire is in order.

The important range of reduced frequencies for aircraft shimmy analysis is the range of $0.1 < \Omega < 2.0$ rad/ft. This

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